

# EXTREMES OF BRANCHING LÉVY PROCESSES

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I will talk recent results on the extremes of supercritical branching Lévy processes  $\{\mathbb{X}_t, t \geq 0\}$ , where the spatial motions are Lévy processes.

In supercritical case, if the Lévy process has regularly varying tails, when properly renormalized,  $\mathbb{X}_t$  converges weakly. As a consequence, we obtain a limit theorem for the order statistics of  $\mathbb{X}_t$ . The result is drastically different from the case of branching Brownian motions.

In critical case, we study asymptotic behaviors of the tails of extinction time and maximal displacement of a critical branching killed Lévy process  $(\mathbb{X}_t^{(0,\infty)})_{t \geq 0}$  in  $\mathbb{R}$ , in which all particles (and their descendants) are killed upon exiting  $(0, \infty)$ . Let  $\zeta^{(0,\infty)}$  and  $M_t^{(0,\infty)}$  be the extinction time and maximal position of all the particles alive at time  $t$  of this branching killed Lévy process and define  $M^{(0,\infty)} := \sup_{t \geq 0} M_t^{(0,\infty)}$ . Under the assumption that the offspring distribution belongs to the domain of attraction of an  $\alpha$ -stable distribution,  $\alpha \in (1, 2]$ , and some moment conditions on the spatial motion, we give the decay rates of the survival probabilities

$$\mathbb{P}_y(\zeta^{(0,\infty)} > t), \quad \mathbb{P}_{\sqrt{t}y}(\zeta^{(0,\infty)} > t)$$

and the tail probabilities

$$\mathbb{P}_y(M^{(0,\infty)} \geq x), \quad \mathbb{P}_{xy}(M^{(0,\infty)} \geq x).$$

In subcritical case, we study asymptotic behaviors of a branching killed Brownian motion with drift  $-\rho$ . Let  $\tilde{\zeta}^{-\rho}$  be the extinction time,  $\tilde{M}_t^{-\rho}$  the maximal position of all the particles alive at time  $t$  and  $\tilde{M}^{-\rho} := \max_{t \geq 0} \tilde{M}_t^{-\rho}$  the all time maximal position. We establish the decay rates of  $\mathbb{P}_x(\tilde{\zeta}^{-\rho} > t)$  and  $\mathbb{P}_x(\tilde{M}^{-\rho} > y)$  as  $t$  and  $y$  tend to  $\infty$  respectively. We also give a Yaglom-type limit theorem.

The talk is based on joint works with Haojie Hou, Yiyang Jiang, Renming Song and Rui Zhang.

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