SEMINARIO ITINERANTE DE TEORÍA DE REPRESENTACIONES
CCM-FC-IM, UNAM
DICIEMBRE 8 Y 9 DE 2019

PROGRAM

Sunday, December 8.

7:00h - 8:50h BREAKFAST AT *TERRAZA ISTMO*

09:00h - 10:00h Nicolle González (UCLA)
Crystals for Demazure modules and nonsymmetric Macdonald polynomials

10:10h - 10:50h Alejandro Argudín (IMUNAM-CDMX)
Yoneda Ext, arbitrary coproducts, and Ab4 categories

10:50h - 11:30h COFFEE BREAK

11:30h - 12:30h Lara Bossinger (IMUNAM-Oaxaca)
Universal coefficients for cluster algebras and Gröbner degenerations

12:45h - 13:45h Jon Wilson (IMUNAM-CDMX)
TBA

14:00h - 15:45h LUNCH AT *TERRAZA ISTMO*

16:00h - 17:00h Yadira Valdivieso (University of Leicester)
Relating the Hochschild cohomology of surfaces algebras and trivial extensions

17:20h - 18:00h Alberto Castillo (IMUNAM-CDMX)
Geometric Construction for Kac-Moody Lie Algebras of Affine Type

19:30h DINNER. LOCATION TBA.

Monday, December 9.

7:00h - 9:50h BREAKFAST AT *TERRAZA ISTMO*

10:10h - 10:50h Mindy Huerta (IMUNAM-CDMX)
Cut cotorsion pairs

10:50h - 11:30h COFFEE BREAK

11:30h - 12:30h José Simental (UC, Davis)
Hilbert schemes on singular curves and rational Cherednik algebras

12:45h - 13:45h Rita Jiménez Rolland (IMUNAM-Oaxaca)
Representation stability and FI-modules

14:00h - 15:45h LUNCH AT *TERRAZA ISTMO*

16:00h - 17:00h Alfredo Nájera (IMUNAM-Oaxaca)
Positivity for cluster algebras is broken line convexity

17:20h - 18:00h Kent Vashaw (Louisiana State University)
Noncommutative tensor triangular geometry

Tuesday, December 10.

7:00h - 9:00h BREAKFAST AT *TERRAZA ISTMO*

Abstracts

Alejandro Argudín. ***Yoneda Ext, arbitrary coproducts, and Ab4 categories.*** Recent generalizations of tilting objects deal with the problem of working with an Ext functor built without projectives or injectives. Such bifunctor, known as the Yoneda Ext, have the benefit that it can be defined in abelian categories without assuming existence of projectives or injectives.

Namely, R. Colpi and K. R. Fuller developed a theory of tilting objects of projective dimension ≤ 1 for abelian categories in [1], and P. Čoupek and J. Šťovíček developed a theory of cotilting objects of injective dimension ≤ 1 for Grothendieck categories in [2]. A fundamental result needed in these theories is that

$$\mathrm{Ext}^1_{\mathcal{A}}\left(\bigoplus_{i\in I} A_i, X\right) = 0 \text{ if and only if } \mathrm{Ext}^1_{\mathcal{A}}(A_i, X) = 0\, \forall i \in I.$$

Such result is proved showing that, in any Ab3 abelian category $\mathcal{A}$, there is an injective correspondence

$$\mathrm{Ext}^1_{\mathcal{A}}\left(\bigoplus_{i\in I} A_i, X\right) \to \prod_{i\in I} \mathrm{Ext}^1_{\mathcal{A}}(A_i, X).$$

Now, for developing a theory of tilting objects of projective dimension $\leq n$, it is needed a similar result for Ext^n. But, it is not known in general if there is an injective correspondence $\mathrm{Ext}^n_{\mathcal{A}}\left(\bigoplus_{i\in I} A_i, X\right) \to \prod_{i\in I} \mathrm{Ext}^n_{\mathcal{A}}(A_i, X)$.

The goal of this talk is to show how to build, in any Ab4 category $\mathcal{B}$, a bijective correspondence $\mathrm{Ext}^n_{\mathcal{B}}\left(\bigoplus_{i\in I} A_i, X\right) \to \prod_{i\in I} \mathrm{Ext}^n_{\mathcal{B}}(A_i, X)\ \forall n > 0$. Furthermore, it will be discussed if the existence of such bijective correspondences characterizes Ab4 categories.

References:

[1]: Colpi, Riccardo and Fuller, Kent, "Tilting objects in abelian categories and quasi-tilted rings", Transactions of the American Mathematical Society 359, 2 (2007), pp. 741–765.

[2]: Čoupek, Pavel and Št'ovı́ček, Jan, "Cotilting sheaves on noetherian schemes", arXiv preprint arXiv:1707.01677 (2017).

Lara Bossinger. ***Universal coefficients for cluster algebras and Gröbner degenerations.*** I will introduce universal coefficients for cluster algebras of finite type with frozen directions (slightly generalizing Fomin and Zelevinsky's original definition). In particular, I will focus on the homogeneous coordinate ring $A_{2,n}$ of the Grassmannain of planes under its Plücker embedding (a cluster algebra of type $\mathtt{A}_{n-3}$ with n frozen directions). Interpreting the coefficients as degeneration parameters, the cluster algebra with universal coefficients is a family of cluster algebras with so-called specified coefficients. The main result I will present is that in the Grassmannian case the family of cluster algebras with specified coefficients corresponds to all Gröbner-degenerations of $A_{2,n}$ given by a specific maximal cone in the Gröbner fan of the Plücker ideal and all its faces. The talk is based on joint work with F. Mohammadi and A. Nájera Chávez.

Alberto Castillo. ***Geometric Construction for Kac-Moody Lie Algebras of Affine Type.*** Let C be the symmetizable generalized Cartan matrix of type $\widetilde{C}_n$, and let $\mathfrak{g} = \mathfrak{g}(C)$ be the Kac-Moody Lie algebra attached to C. Let D be the minimal symmetrizer of C. With this datum together with an orientation Ω of the graph naturally attached to C, we associate a finite-dimensional algebra $H = H(C, D, \Omega)$ defined by a quiver with relations, introduced in [GLS1]. This algebra makes sense over an arbitrary field K, but here we fix $K = \mathbb{C}$ so that varieties of H-modules are complex varieties. By [GLS2] the affine varieties $\mathrm{rep}_{\mathrm{l.f}}(H, \mathbf{r})$ of locally free H-modules with rank vector $\mathbf{r}$ are smooth and irreducible. We associate to H a convolution bialgebra $\mathcal{M} = \mathcal{M}(H)$ of constructible functions on the varieties $\mathrm{rep}_{\mathrm{l.f}}(H, \mathbf{r})$ [GLS3]. In this talk, I will realize the enveloping algebra $U(\mathfrak{n})$ of the positive part of $\mathfrak{g}$ as the convolution algebra $\mathcal{M}$.

Referencias

[GLS1] C. Geiss, B. Leclerc, J. Schröer. *Quivers with relations for symmetrizable Cartan matrices I: Foundations.* Invent. Math. 209 (2017), 61-158.

[GLS2] C. Geiss, B. Leclerc, J. Schröer. *Quivers with relations for symmetrizable Cartan matrices II: Change of Symmetrizers.* Int. Math. Res. Not. IMRN 2018, no. 9, 2866-2898.

[GLS3] C. Geiss, B. Leclerc, J. Schröer. *Quivers with relations for symmetrizable Cartan matrices III: Convolution algebras.* Represent. Theory 20 (2016), 375-413.

Nicolle González. ***Crystals for Demazure modules and nonsymmetric Macdonald polynomials.*** Demazure modules arise as Borel modules generated by extremal weight vectors. The nonsymmetric Macdonald polynomials, introduced by Opdam, Macdonald, and Cherednik, are generalizations that recover the usual symmetric Macdonald polynomials under certain specializations. By work of Ion and Sanderson, these polynomials are intimately tied with Demazure modules since in affine types they can be realized as characters of affine Demazure modules. However, the connections run deeper. Based on joint work with Assaf, I will discuss how by constructing an explicit combinatorial model for finite Demazure crystals one can obtain the nonsymmetric Macdonald polynomials as the character of a graded sum of finite Demazure modules. Moreover, by extending the combinatorial constuction to the setting of affine crystals, it can be shown that any affine Demazure module admits a finite Demazure flag.

Mindy Huerta. ***Cut cotorsion pairs.*** Given two classes of objects in an abelian category, it is not always possible to get that the pair forms a cotorsion pair. In this talk, I will introduce the notion of "Cut cotorsion pair" in order to give a relativization of complete cotorsion pairs. In this case, relativization means that, given a pair of classes of objects in an abelian category, we study the properties that the involved classes have so that we can find a suitable subcategory where the pair works as a complete cotorsion pair on it.

Rita Jiménez Rolland. ***Representation stability and FI-modules.*** In this talk we will describe the category of FI-modules and some of its key properties. Our main goal is to show how FI-modules encode some structural properties that govern certain naturally-arising sequences of S_n- representations. Time permitting, we will discuss how this setting can be applied to deduce strong constrains on the cohomology of the configuration space of n distinct ordered points on connected manifolds and the cohomology of pure mapping class groups of orientable and non-orientable surfaces.

Alfredo Nájera. ***Positivity for cluster algebras is broken line convexity.*** In 2014, Gross, Hacking, Keel and Kontsevich (GHKK) introduced the so-called theta basis for a cluster algebra. The elements of the theta basis -theta functions- are a very special class of functions on a cluster variety. In order to describe theta functions one considers sophisticated objects called scattering diagrams and broken lines inside them. A key insight of the work of GHKK is that theta functions on a cluster varieties play a similar role to the role played by the characters of an algebraic torus in toric geometry. In particular, they showed that one can consider "positive subsets" inside the scattering diagram to compactify a cluster variety. Various projective varieties arising in representation theory (such as flag varieties, Grassmannians, certain Schubert varieties, etc.) fit this framework.

The definition of a positive set is of algebraic nature and is a notion directly linked to the positivity property of cluster algebras. The purpose of this talk is to present a geometric/combinatorial interpretation of positive sets. The main result I will talk about is the following: a set is positive if and only if it is "broken line convex". I will work in a restricted generality so that the main technical gadgets to consider can be described using representation theory of quivers.

This talk is based on joint work with M.-W. Cheung and T. Magee.

José Simental. ***Hilbert schemes on singular curves and rational Cherednik algebras.*** Let n and m be coprime positive integers. We consider the Hilbert scheme of points on the singular curve $C = \{x^n = y^m\}$. This is the moduli space parametrizing ideals of finite codimension inside the algebra of functions on C. As such, it carries a natural torus action that is induced from a torus action on C. We will show that the equivariant homology of the (parabolic) Hilbert scheme carries an action of the gl_n rational Cherednik algebra at the parameter $c = m/n$, and we will explicitly identify this representation as the unique simple quotient of the polynomial representation. This descends to an action of the spherical subalgebra on the homology of the honest Hilbert scheme. Time permitting, I will explain why one expects such an action to exist. Our methods are mostly combinatorial and I won't assume any familiarity with Cherednik algebras or Hilbert schemes. This is joint work with Eugene Gorsky and Monica Vazirani.

Yadira Valdivieso. ***Relating the Hochschild cohomology of surfaces algebras and trivial extensions.*** The notion of the trivial or split extension of an algebra A by a bimodule M has played an important role in several areas of mathematics, like cohomology theory, representation theory, category theory, homological algebra, and cluster theory.

Some homological properties, like global dimension, finitistic projective dimension and Gorenstein properties of the trivial extension $T(A)$ of an algebra A are strongly related to those of A. For instance the first Hochshild cohomology group $\mathrm{HH}^1(\mathrm{T}(A))$ of a trivial extension $\mathrm{T}(A)$ can be computed in some cases in terms of $\mathrm{HH}^1(A)$.

In this talk, I will exhibit a formula to compute the Hochschild cohomology groups in lowers degrees of cluster tilted algebras, which are trivial extension of tilted algebras, and show how these groups are related to those of the surfaces algebras arising from surfaces with boundaries.

Kent Vashaw. ***Noncommutative tensor triangular geometry.*** We describe a general theory of prime, completely prime, and semiprime ideals of non-braided tensor-triangulated categories. These notions are a noncommutative analogue to Paul Balmer's prime spectra of symmetric tensor-triangulated categories. Noncommutative tensor-triangulated categories appear naturally as stable module categories for non-quasitriangular Hopf algebras and as derived categories of bimodules of noncommutative algebras. In stable module categories of Hopf algebras, the support theory of the category, as described by Benson-Iyengar-Krause, is linked to the Balmer spectrum, which is shown to be the final support datum. We will describe how this connection can be used to compute Balmer spectra in general, and we will compute the Balmer spectra for stable module categories of the small quantum group of a Borel subalgebra at a root of unity, and the stable module categories for smash coproduct Hopf algebras of group algebras with coordinate rings of groups.

Jon Wilson. ***TBA.*** TBA

Organizers

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Daniel Labardini (IMUNAM-CDMX)
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